Pricing and Capacity Allocation Strategies of Suppliers in an Electronic Market

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Abstract

We analyze a problem where a market maker on behalf of buyers allocates a given amount of order among \( N \) suppliers each with finite capacity. Specifically, we analyze a situation in which suppliers have the option of going to open market and selling their capacity at a market price. However, the supplier will incur search costs. Moreover, the demand for the supplier capacity in the open market is stochastic. Based on these, we derive the capacity-price curve (supply curve for capacity) for each supplier. The capacity-price curve of the suppliers provides a basis for the market maker to allocate the order among the suppliers so as to minimize his cost. We also identify the key parameters that will influence the overall performance of the electronic market place.

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1. Introduction

Electronic market place for Business-to-Business (B2B) applications primarily plays the role of market making. The Market Maker (MM) brings together fragmented supply and demand and helps allocate supply to meet demand. While MM takes several forms, popular applications include aggregators, exchanges and auctions. Electronic markets promise several benefits to the buyers and open up new opportunities for the suppliers. They bring a large customer (supplier) base to suppliers (customers) by virtue of their ubiquity and low costs of communication. Bakos (1991) argued that electronic markets reduce search costs for the buyer and help her realize better prices. Furthermore, in a differentiated market, the allocation efficiency will also improve. On the other hand, suppliers also benefit from electronic markets in several ways. They experience reduction in customer search costs (Siebel and House, 1999), product promotion costs and transaction costs (Mahadevan, 2000). Because of these advantages, electronic markets are increasingly becoming an attractive channel for procurement. Several studies predicted electronic markets to play a significant role in redefining buyer – supplier relationships (Elmaghraby, 2000, Dai and Kauffman, 2001 and Malone et al., 1987).

Irrespective of the form that an electronic market takes, it serves as an alternative new channel for the buyers and the suppliers to match supply with demand. An electronic market provides several benefits that simply did not exist earlier. We motivate our
research by a brief discussion of the implications of this new channel on industrial procurement practices.

Consider, for instance, Converge (www.converge.com), an electronic market for the semi-conductor industry. Suppliers participating in Converge can offload their excess and unsold capacity and thereby improve utilization. On the other hand, buyers can handle short-term surges in demand and effectively meet ramp-up decisions. Converge plays the role of a market maker (MM), improving supply-demand matching in the short-run.

Although suppliers have an option to sell capacity in both the open market and the electronic market, they face different sets of costs (Grey et al. 2002). Consequently, suppliers will deploy alternative strategies for pricing their capacity in these markets. We believe that business environment aspects such as search cost differentials and risks associated with selling capacity in the open market would influence suppliers’ pricing strategies and electronic market participation and capacity allocation. We focus our attention on this issue and develop a framework to analyze the pricing behaviour of the suppliers in an electronic market.

An MM faces several strategic issues in hosting the market place. What will be the impact of the number of suppliers selected for award of contract from a pool of existing suppliers on the price offered by the suppliers and on MM’s profitability? What is the optimal number of suppliers to award a contract? Can the MM estimate \textit{apriori} the number of suppliers to be selected? We elevate the discussion to a more
general and broader level by addressing these issues pertaining to configuration and operation of electronic markets.

We make three significant contributions through this research. First, we provide a quantitative basis for analyzing the pricing behavior of participating suppliers in an electronic market. Further, we link suppliers’ pricing strategies to MM’s decision making framework thereby addressing compatibility issues in an electronic market. Such an approach is critical to sustenance of the marketplace in the long run. Finally, we derive closed form policy structures for suppliers’ pricing decisions and MM’s decision and develop new insights to the problem. Contrary to the intuitive thinking, we find that a supplier is likely to charge more as the MM contracts for more capacity from the supplier. We also find that under certain conditions, the MM can apriori announce the number of suppliers to be awarded the contract and minimize its cost of the items procured.

The rest of the paper is organized as follows. In the next section we present a review of the related literature. In section 3 we introduce the problem and follow it up with a model for the pricing behavior of the suppliers in section 4. In section 5, we discuss the implications of the current research and develop additional insights and finally conclude the paper in section 6.

2. Literature review

Three streams of research in supplier selection and procurement practices provide relevant literature to our study. One stream pertains to supplier selection literature.
The MM in an electronic market faces the issue of supplier selection. Most approaches deal with the supplier selection problem using price, quality and other factors using either simple weighting scheme of qualitative factors or mathematical programming methods (Vaidyanathan et al. 1999, Narasimhan and Stoynoff, 1986). Weber et al. (1991) provides a review of research on supplier selection methods using these approaches. These approaches do not address the problems arising out of information asymmetry. Moreover, the relationship between capacity and price has not been explicitly considered. In our model, we explicitly model the relationship between price and capacity and its impact on the procurement cost for the buyer. These inter-relationships will govern supplier selection.

Elmaghraby (2000) reviews the trends in the sourcing literature and identifies a central issue of multiple sourcing: the number of vendors to be selected. In particular, the review identifies one of four key questions as being the endogenous determination of the number of suppliers through an auction process. In an early paper, Seshadri et al. (1991) note that multiple sourcing increases the probability of more bidders; which leads to the higher dedicated capacity critical to insure the buyer against surges in demand. They identify the tradeoff between short-term price increases in creating slack capacity in the supplier base and the long-term advantages in supply assurance and price reductions.

The second stream addresses informational aspects of the sourcing problem. Elmaghraby (2000) reports that there is a paucity of papers that incorporate the informational aspects of the sourcing problem. Most approaches treat the supply
relationship as a single-decision maker optimization issue, rather than an interactive vendor – buyer situation. A supply chain, however, rarely is managed by a central planner (ibid.). Sourcing strategies depend on market environments and buyers must adjust them for best results; one size does not fit all situations. Elmaghraby’s review identifies several under-researched areas, among them costs, vendor controls on costs, asymmetry and un-observability of costs, and supplier balance and variability across the supplier base. Our model includes search costs along with manufacturing cost, and addresses incentive compatibility issues that arise from un-observability of costs. We also address the supplier capacity and some degree of stochastic differences across the supplier base.

Peng-Sheng You (2000) deals with a sequential buying process, where purchase of a certain number of units of an item at the lowest total purchasing cost must happen within a given number of time periods. The author develops a dynamic model of pricing policies and search rules for the decision maker. In another paper, Gallien and Wein (2000) examine information asymmetry and incentive compatibility in industrial procurement under capacity constraints. Unlike our model, these approaches allow the buyer to allocate variable quantity to vendors using information from multiple rounds of bidding.

The third stream relates to governing mechanisms for quantity allocation and pricing. Variable quantity allocations have a downside in repeated procurement situations. Klotz and Chatterjee (1996) and Khai Sheang Lee (2000) study learning effects in sequential multiple sourcing arrangements. Their models confirm that unequal splits
would create non-symmetric cost structures in future periods, due to asymmetric learning, which may have detrimental effects on future competition. Therefore, we develop our model assuming equal splits.

Fath and Sarvary (2001) argue with a model where B2B exchanges reduce search costs that the size of the exchange is not zero or infinite, but finite and stable. They find that sellers’ prices may not necessarily decrease with lower search costs; but buyers’ surplus usually increases. They call for further research to explicitly model the actual market mechanisms -- such as auctions, demand aggregation, etc. In a recent paper, Vulcano et al. (2002) analyze multi-period auctions as a tool in revenue management, and propose a list price, capacity-controlled mechanism for setting the opportunity cost of capacity in a single award auction by a single seller. In contrast, our model proposes a multi-award bidding competition among multiple vendors based on endogenously derived price-capacity curves.

In sum, the cited literature demonstrates the need for an explicit incentive compatible model of capacity constrained vendor participation in electronic markets, where the optimal number of vendors may be endogenously determined.

3. Supplier costs and decisions

Before we present our model, we elaborate the problem and formalise the costs and risks faced by a supplier in both the open market and the electronic market. Consider an electronic market hosted by a MM, in which \( n \) suppliers are enlisted. The total amount of capacity the MM is seeking through a reverse auction is denoted by \( \Lambda \). In
reality, $\Lambda$ would represent aggregated requirements of the buyers enlisted in the electronic market as in the case of an aggregator or the requirement of a single buyer as in the case of a reverse auction. In any case, we assume that the MM represents the buyers in the electronic market. The MM would like to allocate $\Lambda$ to a subset of $m$. All $m$ suppliers, finally selected, will get an equal allocation of the capacity at the price of $(m+1)$ lowest supplier’s bid. The fraction of the capacity awarded to each supplier is therefore $\frac{\Lambda}{m}$. This practice of splitting the contract into lots and letting the suppliers know about the number of lots apriori is an important operational feature observed in an electronic market (Rangan, 1999). A preliminary qualification for the supplier to be enlisted by MM is that the supplier’s capacity is at least as large as $\Lambda$. This is to ensure that in case $m = 1$ then there is no capacity infeasibility issues.

Consider the $i^{th}$ supplier participating in the electronic market with a finite capacity $\mu_i$. The supplier faces different sets of costs and risks in the open market and the electronic market. In the open market, the transaction costs are primarily due to search, uncertain demand, and predetermined price levels. To find buyers in the open market the supplier incurs a search cost, $S(x)$, where $x$ is amount of capacity it wants to sell. In general, we assume that, $S(x)$ is increasing in $x$. Search costs include costs that are necessary to conduct business and to find buyers such as costs of advertising, marketing and sales, and other general administrative costs involved in customer acquisition.

While attempting to sell its capacity in the open market, the supplier finds that the demand is stochastic. Hence, the capacity the supplier will be able to sell in the open
market is a random variable $X$. The random nature of demand imposes a burden of financial risk on vendors. Finally, the historical price per unit of capacity in the open market is $P$. We assume the price per unit of capacity is uniform across all suppliers. Such an assumption is common in the literature (see for example Serel et al. 2001).

If the supplier utilizes $x$ units of capacity for production, then the supplier’s production cost is given by $C(x)$. We assume that all suppliers have access to the same production technology in which case the unit production cost is identical for all suppliers. Such an assumption is reasonable given large-scale efforts of suppliers in recent times to acquire similar technology and best practices to remain competitive. The demand distribution for suppliers’ capacity, in the open market, is assumed to be independently and identically distributed and the search cost function is identical across all suppliers. With the advent of Internet technologies, reduction in communication costs have levelled the ground and narrowed down search cost differentials among competing suppliers. We, therefore, assume that the suppliers differ from each other only in terms of capacity.

4. Price – Capacity curve of the suppliers

From the suppliers’ perspective, she will sell her capacity in the open market at a unit price of $P$. However, since she will face uncertain demand and incur search costs, she will consider selling some capacity to MM at a lower price, as long as the price differential between the open market and the electronic market is more than the costs incurred in selling additional capacity in the open market. Based on these
considerations, we derive the price – capacity curve for a supplier to participate in an electronic market.

Consider the case of the $i^{th}$ supplier. Her expected profit if she offers her entire capacity in the open market is given by

$$PE\{\min(X_i, \mu_i)\} - S_i(\mu_i) - C_i(E\{\min(X_i, \mu_i)\})$$

(1)

where $\mu_i$ is the supplier’s capacity.

The first term in the above expression is the expected revenue earned ($X_i$ is the amount of capacity she can sell in the open market), the second term is the search cost incurred and the final term is the production cost. The $E$ in the equation is the expectation operator. The search cost is a function of the number of units of capacity she wishes to sell in the open market. Now suppose the supplier offers $\lambda_i$ units of capacity to MM, in which case she will have to sell $\mu_i - \lambda_i$ units of capacity in the open market. Her expected profit will then be:

$$P_i(\lambda_i) = PE\{\min(X_i, \mu_i - \lambda_i)\} - S_i(\mu_i - \lambda_i) - C_i(\lambda_i + E\{\min(X_i, \mu_i - \lambda_i)\})$$

(2)

In (2), $P_i(\lambda_i)$ is the price the supplier quotes to the MM, which is a function of units of capacity the MM procures from this supplier. The supplier will commit $\lambda_i$ units of capacity provided (2) is at least as large as (1). Therefore the price-capacity curve is given by:

$$P_i(\lambda_i) = \frac{P[E\{\min(X_i, \mu_i)\} - E\{\min(X_i, \mu_i - \lambda_i)\} - \{S_i(\mu_i) - S_i(\mu_i - \lambda_i)\}]}{-\{C_i(E\{\min(X_i, \mu_i)\}) - C_i(\lambda_i + E\{\min(X_i, \mu_i - \lambda_i)\})\}} / \lambda_i$$

(3)
Lemma 1 If the search cost $S(x)$ is increasing and convex and production cost is given by $C(x) = vx, v > 0$, then the unit price quoted by the supplier increases with capacity committed to MM.

The proof is available in the Appendix.

The above lemma is valid for any probability distribution of demand for supplier’s capacity in the open market. At the outset, the result appears counter-intuitive. However, it can be argued that as the supplier commits every additional unit of capacity to the MM, the risk of not being able to sell the remaining capacity in the open market comes down. Furthermore, the search costs also come down, enabling the supplier to charge a premium for the balance capacity. We also provide further analysis later in the paper (in Lemma 3) to formally explain this observed pattern.

However, this pattern introduces additional dimensions and provides more insights into suppliers’ behaviour in electronic markets. The suppliers’ ability to increase price is limited by the historical price $P$, since at this point, the MM may not find enough buyers interested in using the electronic market. As argued earlier, in reality, the upper bound on the price that a supplier will quote in the electronic market is likely to be somewhat lower than $P$ as the supplier will face lower search and customer acquisition costs in an electronic market. Therefore, it will be of interest to know the maximum price likely to be quoted by the suppliers in an electronic market.

Lemma 2 Assume that the demand for capacity in the open market for the supplier is uniformly distributed $(0, b), b > 0$. When $S(x) = kx^2, C(x) = vx, k \geq 0, v > 0$ and $\mu_i \leq b$
then the price quoted by the suppliers increases linearly with capacity. The price is
given by:

\[ P_i(\lambda_i) = P - (2\mu_i - \lambda_i) \left( \frac{P - v}{2b} + k \right) \]  \hspace{1cm} (4)

The proof is available in the Appendix.

The demand distribution parameter \( b \) denotes the maximum available demand for the
supplier. Consider the extreme conditions viz., \( \lambda_i = 0 \) and \( \lambda_i = \mu_i \). In the former, MM
does not procure any capacity (or procures an insignificantly small unit of capacity)
from the supplier and in the latter case, the entire capacity. Substituting these
conditions in (4), we obtain:

When \( \lambda_i = 0 \), \[ P_i(\lambda_i) = P - 2\mu_i \left( \frac{P - v}{2b} + k \right) \]  \hspace{1cm} (5)

and when \( \lambda_i = \mu_i \), \[ P_i(\lambda_i) = P - \mu_i \left( \frac{P - v}{2b} + k \right) \]  \hspace{1cm} (6)

Equations (5) and (6) provide range for the unit price quoted by the supplier to MM.
Moreover, the difference between \( P \) and \( P_i(\lambda_i) \) provides an economic basis for the
existence of MM in the long run. The maximum discount over the open market price
MM gets from the supplier is \( \mu_i \left( \frac{P - v}{2b} + k \right) \) (from equation 6). The result is intuitive
because we find that the discount increases with supplier capacity, contribution
margin \( P - v \) and the search cost parameter \( k \). If the supplier’s mean demand increases
\((0.5b)\) then the discount reduces.
The supplier’s capacity utilization increases when she commits $\lambda_i > 0$ units of capacity to MM. The expected capacity utilization is denoted by $\rho$. Thus we have Lemma 3.

**Lemma 3:** When the supplier commits $\lambda_i$ units of capacity to MM her expected capacity utilization is given by

$$
\rho = \frac{\lambda_i + E[\min(X, \mu_i - \lambda_i)]}{\mu_i} = 1 - \frac{(\mu_i - \lambda_i)^2}{2b\mu_i} \tag{7}
$$

If the supplier sells her capacity only in the open market then her expected utilization is given by $1 - \frac{\mu_i}{2b}$. Her capacity utilization can increase from this point only by selling part of her capacity to MM. As the capacity utilization increases the cost of unsold capacity also increases resulting in higher price quoted by the supplier. Greer and Liao (1986) provided empirical evidence for this phenomenon in the aerospace industry. They reported that suppliers were charging higher price at higher utilization.

As suppliers are identical in terms of production cost and demand distribution (in the open market) the price quoted by the supplier for the capacity sold through the electronic market can be written by substituting $\lambda_i = \frac{\Lambda}{m}$ in the equation (4):

$$
P_i\left(\frac{\Lambda}{m}\right) = P - \left( P - \frac{p - \nu}{2b} + k \right) \left[ 2\mu_i - \frac{\Lambda}{m} \right] \tag{8}
$$

From (8) it is clear that suppliers with larger capacity will offer a lower price. As the mean demand (that is, $0.5b$) for supplier’s capacity increases, the price also increases. This happens because the risk of not being able to sell the entire capacity in the open
market reduces. For a similar reason, when MM’s requirement increases (that is, $A$) then again the price increases.

5. Implications for Electronic Market Operation

Electronic markets serve as a new channel for the buyers and the suppliers to match supply with demand in the short-run. The foregoing analyses show that search cost and risk differentials between traditional open market and the electronic market could potentially induce certain behavioural patterns amongst the suppliers with regard to the price quoted. Furthermore, the price quoted by the suppliers varies on account of their respective capacities. This has significant implications to other operating policies of the electronic market.

We particularly focus our attention on the decision making framework of the MM. While suppliers will have the incentive to participate, how does the MM ensure that it makes the best allocation of its requirement among the participating suppliers? The other issue that the MM confronts is the number of suppliers to be allowed to participate in the process and the subset of those awarded the contract for supply. In typical reverse auction sites such as Freemarkets (www.freemarkets.com), only a certain number of suppliers are pre-qualified to participate in the bidding process. Furthermore, the total requirements are split into lots and the lots are awarded to only a subset of the bidders.

MM’s decision making is complicated because it does not have complete information about participating suppliers. While the MM may be aware of the unit production cost
and the parameters of demand distribution, it may not have knowledge of suppliers’
capacity. We perform additional analysis to develop insights on MM’s decision
process.

5.1. Optimum number of suppliers to allocate the requirement

The salient features of the supplier selection process are as follows. MM pre-qualifies
the number of suppliers (denoted by $n$) to participate in the bidding process, through a
RFQ process, along with reverse auction rules, such as sealed bid, duration of the bid
taking period, and other terms of fulfillment. The MM will also a priori announce the
number of suppliers, $m$ ($0 < m \leq n$) that will be selected. Consequently, the contract
entails an equal amount awarded to the $m$ selected suppliers, with a uniform price
equal to that of the lowest bid price amongst the rejected suppliers. The decision
parameter $m$ will be chosen such that it will minimize the total expected cost of the
items procured through the auction.

The MM obtains quote from the participating suppliers in the form of price – capacity
curve. The price – capacity curve distribution of the participating suppliers would be
the basis for MM to allocate the order among the suppliers so as to minimize the total
expected cost of the cost of the items procured. As MM does not have complete
knowledge of the suppliers’ capacities it assumes that supplier’s capacity is sampled
from a uniform distribution $U(\mu_i, \mu_h)$. We now analyze the problem from the MM’s
perspective by relating the capacity-price curve distribution of the suppliers to its (that
is, MM’s) cost minimization function.
As the MM has to announce the number of suppliers that will be selected (that is, $m$), it will do in a fashion that will minimize its total expected cost of the items procured. MM’s problem can therefore be formulated as:

$$\min_{m \geq 0} \Lambda E(P_{[m+1]}) = \Lambda \min_{m \geq 0} E(P_{[m+1]}),$$

where $P_{[1]} \leq P_{[2]} \leq \cdots \leq P_{[m]} \leq P_{[m+1]} \leq \cdots \leq P_{[n]}$ denotes the order statistics and $P_{[m+1]}$ is the lowest bid price amongst the rejected suppliers.

From (8), it can be shown that $E(P_{[m+1]}) = P - \left( \frac{P - v}{2b} + k \right) \left( 2E(\mu_{[n-m]}) - \frac{\Lambda}{m} \right)$.

Again we use a similar notation for order statistics for the random variable $\mu$, $\mu_{[1]} \leq \mu_{[2]} \leq \cdots \leq \mu_{[m]} \leq \cdots \mu_{[n-1]} \leq \mu_{[n]}$.

Substituting $E(\mu_{[n-m]}) = \mu_t + \frac{n-m}{n+1} (\mu_h - \mu_t)$ in the above equation and simplifying we get an expression for expected unit price to be paid by the selected suppliers.

$$E(P_{[m+1]}) = P - \left( \frac{P - v}{2b} + k \right) \left[ \mu_t + \frac{n-m}{n+1} (\mu_h - \mu_t) \right] - \frac{\Lambda}{m} \right), \tag{9}$$

The market maker’s optimization problem stated above can now be solved by simple calculus and the optimum value of $m$ can be obtained.

**Lemma 4** The optimum number of suppliers that the MM will select is given by

$$m^* = \sqrt{\frac{\Lambda(n+1)}{2(\mu_h - \mu_t)}}.$$

See Appendix for proof.
The expression for optimum number of suppliers exhibits several properties of interest to the MM. It is dependent only on total capacity requirement of MM, the number of participating suppliers and MM’s knowledge of the range of supplier’s capacity. Interestingly, all these parameters are not only known to MM but are also independent of supplier specific parameters related to manufacturing and search costs. The only error that MM can introduce in the computation of \( m^* \) is in the estimation of \( \mu_h \) and \( \mu_l \). Since \( m^* \) is less sensitive to these two parameters (because of the square root factor), the computation is expected to be robust.

5.2 Impact of the number of pre-qualified suppliers

It is obvious from (9) that an increase in the number of suppliers allowed into the bidding process will proportionately push up the value of \( m^* \). What is, however, of interest to the MM is to know the impact of this on its expected winning bid price. A better understanding of this aspect helps the MM in arriving at an appropriate value of \( n \). We now analyze the impact of \( n \), the number of pre-qualified suppliers, on the winning bid price \( E(P_{[m+1]}) \). Substituting the expression \( m^* \) in equation (9) and simplifying we get

\[
E(P_{[m+1]}) = P - 2 \left( \frac{P - v}{2b} + k \right) \left( \mu_l + \frac{n(\mu_h - \mu_l)}{n + 1} - \sqrt{\frac{2\Lambda(\mu_h - \mu_l)}{(n + 1)}} \right)
\]

Equation (10) indicates that the expected winning bid price is decreasing and convex in \( n \). Clearly, as more number of suppliers is allowed to participate in the auction, MM discovers better price.
6. Conclusions

Electronic markets provide a new channel for the buyers and suppliers to match demand with supply in the short run. While suppliers and buyers experience several benefits of utilizing the electronic market place, the operating characteristics and pricing strategies differ from traditional open market. Our model suggests that suppliers would quote a higher price for committing higher levels of capacity in the electronic market. Under certain conditions, MM could *a priori* announce the number of suppliers to be selected for award of contract through a reverse auction mechanism and obtain low costs for the items procured through the contract.

The proposed model could be extended to provide additional insights into the problem. Replacing uniform distribution for demand and capacity variations among the participating suppliers with alternative distributions will be a useful extension of the model. Moreover, at this stage, we have assumed that only the capacity available with each supplier ($\mu_i$) varies. However, in reality the manufacturing cost parameter ($\nu$) also could vary. For instance, Asian suppliers will have a different cost structure compared to that of European and US suppliers and could have a much lower price – capacity threshold. Incorporating this aspect will allow us to assess the impact of international supplier participation typical to an electronic market. Further, we have not modeled explicitly any cost to pre-qualify the suppliers. However, in reality MM often incurs significant costs in pre-qualifying the suppliers. Incorporating these costs into our analysis will yield a different structure and perhaps will limit the number of suppliers to be pre-qualified.
References


Appendix

Proof of Lemma 1: If $X$ is a non-negative random with density function $f(x)$ and distribution function $F(x)$, and $\xi$ is a constant then it can be shown that

$$E\{\min(\xi, X)\} = \xi(1 - F(\xi)) + \int_0^\xi xf(x)dx$$  \hspace{1cm} (A1)$$

To prove Lemma 1 we need to show that $\frac{\partial P(\lambda_i)}{\partial \lambda_i} \geq 0$, where $P(\lambda_i)$ is given by equation (3). We suppress the supplier’s index $i$ for brevity.

$$P(\lambda) = \frac{(P-v)\left\{E\{\min(X, \mu) - \min(X, \mu - \lambda)\} \right\}}{\lambda} - \frac{S(\mu) - S(\mu - \lambda)}{\lambda}. \text{ We consider the first expression on the rhs. Ignoring the (P-v) which is a positive term (otherwise the supplier will go out of business), it suffices to show that the function}$$

$$G(\lambda) = \frac{E\{\min(X, \mu) - \min(X, \mu - \lambda)\}}{\lambda} \text{ is an increasing function of } \lambda. \text{ Substituting (A1) in } G(\lambda) \text{ we get}$$

$$G(\lambda) = \frac{\mu[1 - F(\mu)] + \int_0^\mu xf(x)dx - (\mu - \lambda)[1 - F(\mu - \lambda)] - \int_0^{\mu - \lambda} xf(x)dx}{\lambda} \hspace{1cm} (A2).$$

$$\frac{\partial G(\lambda)}{\partial \lambda} = -\frac{\mu[1 - F(\mu)]}{\lambda^2} - \frac{\int_0^\mu xf(x)dx}{\lambda^2}$$

$$+ \frac{\lambda[1 - F(\mu - \lambda)] - \lambda(\mu - \lambda)f(\mu - \lambda) + (\mu - \lambda)[1 - F(\mu - \lambda)]}{\lambda^2}$$

$$+ \frac{\int_0^{\mu - \lambda} xf(x)dx + \lambda(\mu - \lambda)f(\mu - \lambda)}{\lambda^2}$$

After algebraic simplification we get the expression below:
\[
\frac{\partial G(\lambda)}{\partial \lambda} = -\int_0^\mu xf(x)dx + \mu F(\mu) + \int_0^{\mu \lambda} xf(x)dx - \mu F(\mu - \lambda)
\]
\[
\frac{\lambda^2}{\int_0^\mu (\mu - x)f(x)dx - \int_0^{\mu \lambda}(\mu - x)f(x)dx}
\]

The second expression on the rhs is \( H(\lambda) = -\frac{S(\mu) - S(\mu - \lambda)}{\lambda} \). To show that this is an increasing function we first need a theorem from Bazaraa and Shetty (1979).

**Theorem** (Bazaraa and Shetty, Chapter 3, page 91): let \( S \) be a non-empty open convex set in \( E_n \) and let \( f : S \to E_1 \) be differentiable on \( S \). Then \( f \) is convex if and only if for any \( \bar{x} \in S \), we have
\[
f(x) \geq f(\bar{x}) + \nabla f(\bar{x})'(x - \bar{x}) \quad \text{for each } x \in S . \tag{A3}
\]
\[
\frac{\partial H}{\partial \lambda} = -\frac{\lambda S'(\mu - \lambda) - S(\mu) + S(\mu - \lambda)}{\lambda^2} . \quad \text{As } S(.) \text{ is increasing and convex then from inequality (A3) it is straightforward to show that } \frac{\partial H}{\partial \lambda} > 0 .
\]
As both \( G(\lambda) \) and \( H(\lambda) \) are increasing, therefore \( P(\lambda) \) is also increasing.

**Proof of Lemma 2**: Let us assume that \( f(x) \) is a Uniform \((0,b)\) where \( b > 0 \). Then after simplification we get
\[
E\{\min(\xi, X)\} = \frac{2b^2 - \xi^2}{2b}, \quad 0 \leq \xi \leq b
\]
\[
= \frac{b}{2}, \quad \xi > b \tag{A4}.
\]
Substituting (A4) in (3) we get after simplification equation (4).

**Proof of Lemma 4**: MM minimizes her cost by minimizing the expected bid price.

The expected bid price is given by equation (9). The optimality condition is given by
\[ \frac{dE(P_{(m+1)})}{dm} = 0. \] Thus we have
\[
\frac{dE(P_{(m+1)})}{dm} = -\frac{\Lambda}{m^2} \left( \frac{P-v}{2b} + k \right) + \frac{B}{n+1} (\mu_h - \mu_l) = 0,
\]

where
\[
B = \left[ \frac{P-v}{b} + 2k \right].
\]

Solving for optimum \( m \) we get
\[
m^* = \sqrt{\frac{\Lambda \left( \frac{P-v}{2b} + k \right) (n+1)}{\left( \frac{P-v}{b} + 2k \right) (\mu_h - \mu_l)}} = \sqrt{\frac{\Lambda (n+1)}{2(\mu_h - \mu_l)}}.
\]