IIMB-WP N0. 580

WORKING PAPER NO: 580

Influencer Marketing with Fake Followers

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Year of Publication – February 2019

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Abstract

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The optimal contract stipulates a fixed payment equal to the influencer's outside option and a variable payment increasing in her follower count. In the pre sign-up scenario, the advertiser extracts all the surplus and the equilibrium features truthful display of the influencer's follower count. However in the post sign-up scenario, the advertiser must pay over and above the influencer's outside option; and needs to tolerate high levels of faking. Our results suggest that advertisers are better o_ hiring potential influencers with social media-independent mass appeal rather than the more common practice of hiring them based on merely their follower count.

Keywords: Digital marketing, social media, influencer marketing, fake followers, optimal control, contract theory.

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JEL Classification: D82, D86, M31, M37

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We thank Bruno Badia, Manaswini Bhalla, Pradeep Chintagunta, Tirthatanmoy Das, Sreelata Jonnalagedda, Ashish Kumar, Sanjog Misra and participants at the Chicago Booth-India Quantitative Marketing Conference (2018) at IIM Bangalore for their feedback and comments.

"At Unilever, we believe influencers are an important way to reach consumers and grow our brands. Their power comes from a deep, authentic and direct connection with people, but certain practices like buying followers can easily undermine these relationships." — Keith Weed, Chief Marketing and Communications Officer, Unilever (Stewart, 2018)

1 Introduction

Advertisers often pay popular social media users known as *influencers* to endorse their products online. Many of these influencers have large numbers of self-selected followers who share their interests (travel, cooking, etc.), looking up to them for advice in these domains. According to The Economist (2016), YouTube influencers with over 7 million followers command upto $300,000 per sponsored post, while the corresponding figures for Instagram, Facebook and Twitter are $150,000, $187,500 and $60,000 respectively, allowing social media followings to be monetized lucratively. Even influencers with less than 250,000 followers can make hundreds of dollars per sponsored post. A Linqia (2018) survey across sectors including consumer packaged goods, food and beverage and retail in the US finds that 86% of marketers surveyed used some form of influencer marketing in 2017, and of them, 92% reported finding it effective. 39% of those surveyed planned to increase their influencer marketing budgets. Similar trends reported by eMarketer (2017) and IRI (2018) suggest that influencer marketing is growing.

Influencer marketing has led to the emergence of shady businesses called *click farms* which for a price, offer influencers fake followers, inflate the number of "likes" on their fan pages, and post spurious comments on their posts. Influencers use these services to fraudulently command higher fees from advertisers for promotional posts. A New York Times exposé (Confessore et al., 2018) finds that several personalities including social media influencers have bought fake followers from a click farm called Devumi.

Sway Ops, an influencer marketing agency estimates the total magnitude of influencer fraud to be about $1 billion (Pathak, 2017). They find that in a single day, of 118,007 comments sampled on #sponsored or #ad tagged Instagram posts, less than 18% were made by genuine users. Another study by the Points North Group finds that influencers hired by Ritz-Carlton have 78% fake followers (Neff, 2018). The corresponding numbers for Procter and Gamble's Pampers and Olay brands are 32% and 19% respectively. The quote at the beginning of this paper, by Unilever's Chief Marketing and Communications Officer at the Cannes film festival, indicates that marketers are acutely aware of the fake follower problem.

Paquet-Clouston et al. (2017) report that click farm clients pay an average of $49 for every 1,000 YouTube followers. The corresponding figures are $34 for Facebook, $16 for Instagram and $15 for Twitter. Average prices for 1,000 likes on these platforms are $50, $20, $14 and $15 respectively. Google searches corroborate these claims—hundreds of click farms and millions of fake followers are accessible to anyone with a credit card. A Buzzfeed investigation suggests increasing sophistication of fake follower bots; shell companies that calibrate bots based on the behavioral patterns of genuine users (and thus virtually impossible to detect) on their own apps may have siphoned off millions of dollars from advertisers (Silverman, 2018).

In this paper, we design an optimal contract between an advertiser and an influencer such that the advertiser's profits are maximized subject to the influencer's own participation constraint when she can inflate her follower count. In our model, there is one advertiser and one influencer, both of whom are risk neutral. The advertiser proposes to pay the influencer in exchange for brand endorsement to reach her followers on a social media platform. The advertiser can only observe the publicly displayed follower count of the influencer on the social media platform and not her true number of followers. The influencer may inflate her follower count by buying costly fake followers in order to command more payment from the advertiser. We develop optimal contracts for two classes of influencers:

1. A "pre sign-up" influencer, i.e., a potential influencer who does not yet have a social media account but may be incentivized to open one given the expectation of a certain follower count and,

2. A "post sign-up" influencer who is already on social media, and has private information about her true follower count, but may display an inflated number to the advertiser.

We show that in the former case, the optimal contract lets the advertiser pay a fixed sum equal to the influencer's outside option, who in turn has no incentive to buy fake followers. In the latter case, we show that under the optimal contract, the influencer buys fake followers and the advertiser pays over and above her outside option to utilize her services.

From the perspective of the advertiser the pre sign-up case is better than the post sign-up case. This is so because the advertiser is able to extract the full surplus in the former and limits the payment to the influencer at her outside option value. Additionally, the advertiser is able to elicit truthfulness and there is no incentive for the influencer to inflate her follower count. In the post sign-up case however, not only does the advertiser need to pay the influencer a variable payment (increasing in follower count) over and above her outside option; but needs to tolerate high levels of faking too. Therefore, for advertisers to reach their audience most cost-effectively, it is better to induce potential social media influencers to sign-up; then restrict their payment to the monetary value associated with their outside option; and finally extract the benefits of their brand endorsements.

2 Literature survey

We briefly outline three streams of literature germane to our problem: influencer marketing, models of economic fraud and applications of contract theory to marketing. Because some of these domains are large, we outline only a few studies in each, to motivate and contextualize our own work.

2.1 Influencer marketing

While the literature on social influence and opinion leadership is vast, we focus here only on papers discussing influencer marketing explicitly. Hu et al. (2015), in an analytical setup, find that a monopolist hiring a small number of influencers can diminish unpredictability of sales. In the empirical domain, studies like Jin and Phua (2014), Djafarova and Rushworth (2017) and De Veirman et al. (2017) find that non-traditional celebrities with high follower counts are perceived as more likeable and credible. They significantly influence consumers' product involvement and buying intentions. These studies provide some early evidence on the efficacy of influencer marketing, explaining why advertisers base influencer payments on follower counts. Our own contribution to the influencer marketing literature is to investigate the economics of rampant fake follower fraud, a phenomenon that has received widespread media, but little academic attention thus far.

2.2 Economic fraud

Online businesses are prone to several kinds of marketing fraud. Advertisers paying per click frequently encounter click fraud where click farms simulate genuine clicks. Wilbur and Zhu (2009) in an important, related study investigate the problem of click fraud in search advertisement in a game theoretic setting. Their results suggest that usage of a neutral third party to audit click fraud detection can benefit the search advertising industry. Another form of online fraud consists of fake reviews, when businesses post either fake positive reviews for themselves or fake negative reviews for their competitors. Lappas et al. (2016) demonstrate how even a few fake reviews can significantly boost hotels' visibility. Luca and Zervas (2016) find that the prevalence of suspicious restaurant reviews on Yelp has grown over time. They find that restaurants with weaker reputations tend to engage more in online review fraud when faced with increasing competition.

A model of insurance and sharecropping fraud where agents involve in costly falsification is developed in a contract theoretic setting in Crocker and Morgan (1998).[1] Their model yields results that have been extended to other fraud scenarios, like misreporting of earnings by CEOs (Crocker and Slemrod, 2007; Sun, 2014), many types of insurance fraud (Crocker and Tennyson, 2002; Dionne et al., 2009; Doherty and Smetters, 2005), and in designing optimal product return policies (Crocker and Letizia, 2014).

Employee theft in retail can also be modelled analytically. Mishra and Prasad (2006) demonstrate that a complete elimination of theft may be economically infeasible; they derive an optimal frequency of random inspections to minimize losses due to theft by retail employees. With this paper, we contribute to the literature on economic fraud, by modeling the emerging phenomenon of fraud in influencer marketing; and showing that eliminating fraud may be impossible even under optimal contracts.

2.3 Contract theory

In contract theory a principal wishes to hire an agent and designs an optimal contract which maximizes its profit while respecting the agent's participation and incentive compatibility constraints.[2]

Contract theoretic approaches have been used in marketing scenarios such as designing warranties and extended service contracts (Padmanabhan and Rao, 1993) and delegation of pricing decisions to salespersons (Bhardwaj, 2001; Mishra and Prasad, 2004, 2005) to name a few. Other noteworthy applications of contract theory in marketing include explaining product development incentives (Simester and Zhang, 2010) and to explain how internal lobbying by salespersons for lower prices can elicit truthful information about market demand (Simester and Zhang, 2014). Our paper incorporates contract

[1] This approach assumes that the agent can falsify claims without any way for the principal to verify. Our approach is primarily based on the Crocker and Morgan (1998) formulation.

[2] See Bolton and Dewatripont (2005) for a comprehensive exposition.

theory and optimal control theory in the digital marketing literature, illustrating the economics of influencer marketing fraud.

3 The model

Consider a risk-neutral advertiser who wishes to reach the followers of a risk neutral influencer. The advertiser proposes to pay the influencer for brand endorsement via social media posts. The influencer privately knows her own number of followers n, and can inflate her follower count by buying fake followers. The advertiser observes only the publicly displayed follower count and not the true number of followers. However, the advertiser is aware that the true number of followers of the influencer are distributed in $[n_L, n_H]$ according to the probability density $f(n)$.[3]

The advertiser's profit is denoted by $\Pi(v_1(n); v_2; u(n))$ and the influencer's payoff is denoted by $Y(v_1(n); v_2; u(n); n)$. Under the optimal contract between the two, the equilibrium outcome is characterized by a 3-tuple: $\{v_1, v_2, u\}$ where $v_1(n)$ is a variable payment depending on the influencer's follower count; v_2 is a fixed payment; and $u(n)$ is the function used by the influencer to inflate her true follower count n.[4]

We note that while the variable payment depends on influencer's publicly displayed, possibly inflated number of followers, i.e., $v_1(u(n))$, the revelation principle (Myerson, 1979) guarantees that the same equilibrium outcome can be achieved under an incentive-compatible direct mechanism where the influencer receives variable payment $v_1(n)$.

In order for the equilibrium to be incentive compatible, it must be that at the optimal v_1^*, v_2^*, u^*, there is no incentive for the influencer to not act according to her own type. This happens only if:

$$Y(v_1^*(n); v_2^*; u^*(n); n) \geq Y(v_1^*(\bar{n}); v_2^*; u^*(\bar{n}); n) \qquad \forall \bar{n} \neq n \in [n_L, n_H]$$

For brevity we denote the value function $Y^*(n) \equiv Y(v_1^*; v_2^*; u^*; n)$ and note that since $Y^*(\cdot)$ is optimal,

$$\frac{\partial Y^*}{\partial v_1} = \frac{\partial Y^*}{\partial v_2} = \frac{\partial Y^*}{\partial u} = 0$$

Using the envelope theorem, we establish the dependence of the optimal value function Y^* on n by:

$$\frac{dY^*}{dn} = \frac{\partial Y^*}{\partial n}$$

3.1 The optimization program

The advertiser wishes to maximize its expected profit:

$$\max_{v_1, v_2, u} \left(\int_{n_L}^{n_H} \Pi(v_1(n); v_2; u(n)) f(n) dn \right)$$

subject to the incentive compatibility constraint:

[3] While n is a natural number we assume hereon for the sake of mathematical convenience that $n \in \mathbb{R}_+$.

[4] We assume that n is not a decision variable and the profit of the advertiser depends only on functions v_1, v_2 and u.

$$\frac{dY}{dn} = \frac{\partial Y}{\partial n}$$

and the participation constraint which in general could be of the following two types: pre sign-up or post sign-up.

3.1.1 Pre sign-up participation constraint

Suppose a potential influencer has not yet signed up on social media. In order to ensure that it is worthwhile for her to participate by signing up and then endorsing the advertiser's product, it must be that the ex-ante expected payoff from participation is more than her outside option $\bar{Y}$:

$$\int_{n_L}^{n_H} Y(v_1(n); v_2; u(n); n) f(n) dn \geq \bar{Y}$$

3.1.2 Post sign-up participation constraint

In this case the influencer is already on social media and privately knows her true number of followers. In order for her to find participation worthwhile, it must be that her realized payoff from n followers is higher than her outside option $\bar{Y}$. Hence the post sign-up participation constraint is:

$$Y(v_1(n); v_2; u(n); n) \geq \bar{Y}$$

4 The optimal control problem

Optimization programs featuring integrals in objective functions and derivatives in the constraints can be solved by setting up an optimal control problem and finding the stationary points of the associated Hamiltonian.

4.1 Pre sign-up optimal control

The pre sign-up optimal control problem is:

$$\max_{v_1(n), v_2, u(n)} \left(\int_{n_L}^{n_H} \Pi(v_1(n); v_2; u(n)) f(n) dn \right) : \tag{1}$$

$$\frac{dY}{dn} = \frac{\partial Y}{\partial n} \tag{2}$$

$$\int_{n_L}^{n_H} Y(v_1(n); v_2; u(n); n) f(n) dn \geq \bar{Y} \tag{3}$$

The expected profit function (1) under the incentive compatibility constraint (2) and pre sign-up constraint (3) can be combined into the following Hamiltonian:

$$\mathbb{H} = \Pi(v_1(n); v_2; u(n)) f(n) + \lambda(n) Y_n + \mu Y(v_1(n); v_2; u(n); n) f(n) \tag{4}$$

In the above Hamiltonian formulation, $Y(\cdot)$, the influencer's payoff function is the state variable with its equation of motion represented by condition (2). The control variable is $u(\cdot)$; $\lambda(n)$ is the co-state variable corresponding to the incentive compatibility contraint (2); and μ is the Lagrangian multiplier associated with the pre sign-up participation contraint (3). The necessary first order conditions are obtained from the Pontryagin maximum principle as below:

Pontryagin conditions

1. Optimality condition:
$$\max_u \mathbb{H} \qquad \forall n \in [n_L, n_H]$$
2. Equation of motion for state:
$$\frac{dY}{dn} = \frac{\partial \mathbb{H}}{\partial \lambda} = Y_n$$
3. Equation of motion for costate:
$$\frac{d\lambda}{dn} = -\frac{\partial \mathbb{H}}{\partial Y}$$
4. Transversality condition for state:
$$\lambda(n_H) = 0$$

Together, the four conditions stated above must be necessarily true at the optimal. Proposition 1 characterizes the solution further.

Proposition 1. *The necessary conditions which characterize the solution of the optimal control problem with the pre sign-up participation constraint are as follows:*

$$f \cdot \left(\Pi_u - \Pi_{v_1} \frac{Y_u}{Y_{v_1}}\right) + \lambda \cdot \left(Y_{u,n} - Y_{v_1,n} \frac{Y_u}{Y_{v_1}}\right) = 0 \tag{5}$$
$$\dot{\lambda} = \frac{d\lambda}{dn} = -f \cdot \frac{\Pi_{v_1}}{Y_{v_1}} - \lambda \cdot \frac{Y_{v_1,n}}{Y_{v_1}} - \mu f \tag{6}$$
$$\int_{n_L}^{n_H} \left(\Pi_{v_2} + \mu \cdot Y_{v_2}\right) f(n) dn = 0 \tag{7}$$

Proof. See appendix A. □

4.1.1 Pre sign-up optimal contract

So far we have not assumed much about the payoff functions of the advertiser or the influencer apart from their risk neutrality. We now discuss the payoff functions of the advertiser and the influencer.

Advertiser's payoff

The advertiser must pay the influencer a sum of (dollars, say) $v_1(n) + v_2$. On the other hand by exposing n followers of the influencer to its endorsement, it earns a sum of $A(n)$, where $A(\cdot)$ is an exogenous revenue function which according to the advertiser captures the benefits (in dollar terms) of reaching out to n social media followers. The advertiser's profit function is the difference between the revenue from reaching the influencer's n followers and the payment made to the influencer.[5]

$$\Pi(v_1(n); v_2; u(n)) = A(n) - v_1(n) - v_2$$

Influencer's payoff

For the influencer, payoff is gained due to payments from the advertiser but there is a cost of inflating follower count.[6] We assume that the cost function varies with the degree of misrepresentation: $c(u(n)-n)$. We assume that $c \geq 0$; $c(0) = 0$; $c'(0) = 0$ and $c'' > 0$. This means that costs are at least zero, no inflation entails no cost and the costs of fraud rise progressively higher as the extent of cover-up increases. Thus the payoff of the influencer is given by:

$$Y(v_1(n); v_2; u(n); n) = v_1(n) + v_2 - c(u(n) - n)$$

Proposition 2 characterizes the pre sign-up optimal contract.

Proposition 2. *The pre sign-up optimal contract is characterized by the following 3-tuple:*

$$u(n) = n \tag{8}$$

$$v_1(n) = 0 \tag{9}$$

$$v_2 = \bar{Y} \tag{10}$$

Proof. See appendix B. □

The optimal contract stipulates that the influencer be paid a fixed amount equal to her outside option. Under this payment rule, the influencer has no incentive to inflate her follower count.

4.2 Post sign-up optimal control

The following Hamiltonian $\mathbb{H}$ captures the post-sign up optimal control problem:

$$\mathbb{H} = \Pi(v_1(n); v_2; u(n))f(n) + \lambda(n)Y_n + \mu(Y(v_1(n); v_2; u(n); n) - \bar{Y})$$

As before, the necessary first order conditions are obtained from the Pontryagin maximum principle. These yield the following two conditions from proposition 1:

$$f \cdot \left(\Pi_u - \Pi_{v_1}\frac{Y_u}{Y_{v_1}}\right) + \lambda \cdot \left(Y_{u,n} - Y_{v_1,n}\frac{Y_u}{Y_{v_1}}\right) = 0$$

$$\dot{\lambda} = \frac{d\lambda}{dn} = -f \cdot \frac{\Pi_{v_1}}{Y_{v_1}} - \lambda \cdot \frac{Y_{v_1,n}}{Y_{v_1}} - \mu f$$

[5] We note that it may not be feasible to have an exact formulation of $A(n)$, as attribution and measurement of returns to advertising may be imprecise (Lewis and Rao, 2015). We note that $A(n)$ is exogenous to v_1, v_2, u, thus converting the profit maximization problem to cost minimization for the advertiser. $A(n)$ could be based on the advertiser's past experience and managerial judgment.

[6] The cost of inflating follower count incorporates both the monetary cost of buying fake followers as well as the cost of covering up that fraud.

Proposition 3 characterizes the post sign-up optimal contract.

Proposition 3. *The post sign-up optimal contract is characterized by the following:*

$$\frac{c'(u-n)}{c''(u-n)} = \frac{F(n)}{f(n)} \tag{11}$$

$$u(n) > n \quad \forall n \in (n_L, n_H] \text{ and } u(n_L) = n_L \tag{12}$$

$$v_2 = \bar{Y} \text{ and } v_1(n) = c(u(n) - n) + \int_{n_L}^{n} c'(u(t) - t)dt \tag{13}$$

Proof. See appendix C. □

Post-sign up optimal conditions stipulate that the advertiser pay a fixed sum equaling the influencer's outside option value; and a variable sum that increases in n according to equation (13). Moreover as equation (12) suggests, there are high levels of fake follower buying and the influencer overstates her number of followers.

While proposition 3 characterizes necessary conditions at optimality, for sufficiency, it is enough that $Y_n > 0$ which in turn guarantees that the post sign-up participation constraint binds only at $n = n_L$ leading to $Y(n_L) = \bar{Y}$; and $Y(n) > \bar{Y} \quad \forall n > n_L$. This is ensured if $c' > 0$.

For implementability we need to ensure that $u' > 0$. For a quadratic cost function of buying fake followers, this is satisfied if $\frac{d}{dn}\big(F(n)/f(n)\big) > 0$.

4.3 Illustration: Quadratic costs of faking

Consider the class of quadratic cost functions:

$$c(u-n) = \alpha \cdot (u-n)^2 + \beta \cdot (u-n) + \gamma$$

From the conditions imposed on this cost function: $c(0) = 0, c'(0) = 0, c'' > 0$, the only admissible quadratic cost functions are:

$$c(u-n) = \alpha \cdot (u-n)^2, \qquad \alpha > 0$$

From proposition 3, equation (11) yields:

$$u(n) = n + \frac{F(n)}{f(n)}$$

If $n \sim U[n_L, n_H]$, then

$$u(n) = 2n - n_L$$

The variable payment is then:

$$v_1(n) = \alpha(u-n)^2 + \int_{n_L}^{n} c'(u(t)-t)dt$$
$$v_1(n) = \alpha((2n-n_L)-n)^2 + 2\alpha \int_0^{n-n_L} zdz$$
$$v_1(n) = 2\alpha(n-n_L)^2$$

Thus the inflation function is affine and increases with n; and the variable payment assumes a quadratic functional form.

5 Discussion

Given the analytical results in the previous section, we now discuss some important observations.

5.1 The extent of fraud

In the post sign-up illustration in section 4.3, it is interesting to note that for any influencer with true follower count $n > (n_L + n_H)/2$, the displayed number of followers $u(n) > n_H$. The advertiser knows that the follower count is egregiously false, yet needs to tolerate it. Such results have also been noted in models of sharecropper fraud where agents with low crop yields report absurdly low crop sizes to share with the principal (Maggi and Rodriguez-Clare, 1995; Crocker and Morgan, 1998). In particular, for quadratic costs and uniform distribution, the optimal inflation function is affine and increasing in n which implies that influencers with higher follower counts fake more.[7]

5.2 The influencer's outside option

To induce participation, the advertiser must match the influencer's outside option. Our results suggest that independent validation of outside option gives the advertiser a better estimate about the true popularity of the influencer. The measure of the outside option is thus best determined by rigorous market research that is independent of an assessment of the influencer's social media follower count.

5.3 Concluding remarks

Our work addresses an important concern facing digital marketers today: the fake follower problem its implications for influencer marketing. We model the problem in a contract-theoretic manner, demonstrating that the influencer can be curtailed from purchasing fake followers in a pre sign-up, but not post sign-up setting. In the latter case, we find that influencers with higher genuine follower counts will buy more fake followers, and the optimal contract must account for this.

Note that our model does not take into account the possibility that buying fake followers could attract more genuine followers; we suggest this as a potential topic for further investigation. We also do not account for innately honest influencers who will never buy fake followers irrespective of incentives to do so, in the manner of Mishra and Prasad (2006).

[7] Confessore et al. (2018) reveal that the list of fake follower buyers include prominent personalities.

Appendices

A Proof of Proposition 1

We recall the first order Pontryagin conditions:

1. Optimality condition:
$$\max_u \mathbb{H} \qquad \forall n \in [n_L, n_H]$$
2. Equation of motion for state:
$$\frac{dY}{dn} = \frac{\partial \mathbb{H}}{\partial \lambda} = Y_n$$
3. Equation of motion for costate:
$$\frac{d\lambda}{dn} = -\frac{\partial \mathbb{H}}{\partial Y}$$
4. Transversality condition for state:
$$\lambda(n_H) = 0$$

A.1 Optimality condition

Since the control function is $u(\cdot)$ the derivative of $\mathbb{H}$ with respect to $u(\cdot)$ must be 0, yielding:

$$\frac{d\Pi}{du} \cdot f + \lambda \cdot \frac{dY_n}{du} + \mu \cdot f \frac{dY}{du} = 0 \tag{14}$$

We note that:

$$\frac{d\Pi}{du} = \Pi_{v_1} \frac{\partial v_1}{\partial u} + \Pi_u$$

and

$$\frac{dY_n}{du} = \frac{\partial Y_n}{\partial v_1} \cdot \frac{\partial v_1}{\partial u} + \frac{\partial Y_n}{\partial u} \cdot 1$$

and

$$\frac{dY}{du} = \frac{\partial Y}{\partial v_1} \cdot \frac{\partial v_1}{\partial u} + \frac{\partial Y}{\partial u} \cdot 1$$

Reinserting the above in (14):

$$f \cdot \left(\Pi_{v_1} \cdot \frac{\partial v_1}{\partial u} + \Pi_u \right) + \lambda \cdot \left(Y_{v_1,n} \cdot \frac{\partial v_1}{\partial u} + Y_{u,n} \right) + f \cdot \mu \cdot \left(Y_{v_1} \cdot \frac{\partial v_1}{\partial u} + Y_u \right) = 0$$

Additionally, at the optimal, since $\frac{dY}{dn} = \frac{\partial Y}{\partial n}$

$$\frac{\partial Y}{\partial v_1} \cdot \frac{dv_1}{dn} + \frac{\partial Y}{\partial u} \cdot \frac{du}{dn} = 0$$

leading to:

$$\frac{\partial v_1}{\partial u} = -\frac{Y_u}{Y_{v_1}}$$

We now express the optimality condition for the pre sign-up optimal control problem as:

$$f \cdot \left(\Pi_u - \Pi_{v_1} \frac{Y_u}{Y_{v_1}}\right) + \lambda \cdot \left(Y_{u,n} - Y_{v_1,n} \frac{Y_u}{Y_{v_1}}\right) = 0 \tag{15}$$

A.2 Equation of motion for costate

$$\frac{d\lambda}{dn} = -\frac{\partial \mathbb{H}}{\partial Y}$$

Consider the right hand side:

$$\frac{\partial \mathbb{H}}{\partial Y} = \frac{\partial \mathbb{H}}{\partial v_1} \cdot \frac{\partial v_1}{\partial Y} + \frac{\partial \mathbb{H}}{\partial v_2} \cdot \frac{\partial v_2}{\partial Y} + \frac{\partial \mathbb{H}}{\partial u} \cdot \frac{\partial u}{\partial Y}$$

Since v_2 does not vary with Y its partial derivative is 0. Additionally, from the optimal condition the last term is 0 as well. Thus

$$\frac{\partial \mathbb{H}}{\partial Y} = \frac{\partial v_1}{\partial Y} \cdot \frac{\partial \mathbb{H}}{\partial v_1}$$

Expanding $\frac{\partial \mathbb{H}}{\partial v_1}$ and substituting it back in the equation of motion for costate, we get:[8]

$$\dot{\lambda} = \frac{d\lambda}{dn} = -f \cdot \frac{\Pi_{v_1}}{Y_{v_1}} - \lambda \cdot \frac{Y_{v_1,n}}{Y_{v_1}} - \mu f \tag{16}$$

A.3 Optimal fixed payment

By definition, at the optimal $v_2 = v_2^*$ is fixed and hence if we consider a modified optimal control formulation without the state equation as constraint, v_2^* remains optimal:

$$v_2^* = \arg\max_{v_2} \left(\int_{n_L}^{n_H} \Pi(v_1; v_2; u) f(n) dn\right)$$
$$\text{subject to}$$
$$\int_{n_L}^{n_H} Y(v_1; v_2; u; n) f(n) dn \geq \bar{Y}$$

The first order condition for optimal v_2 yields:

$$\frac{d}{dv_2}\left(\int_{n_L}^{n_H} \Pi(v_1, v_2, u) \cdot f(n) dn + \mu \cdot \left(\int_{n_L}^{n_H} Y(v_1, v_2, u, n) f(n) dn - \bar{Y}\right)\right) = 0$$

This may be simplified via the Liebniz Integration Rule: $\frac{d}{dx}\int_a^b f(x,t)dt = \int_a^b \frac{\partial}{\partial x} f(x,t)dt$ to yield the third equation:

$$\int_{n_L}^{n_H} \left(\Pi_{v_2} + \mu \cdot Y_{v_2}\right) f(n) dn = 0 \tag{17}$$

[8] Assuming $\frac{\partial Y}{\partial v_1} \neq 0$.

B Proof of Proposition 2

B.1 The optimal inflation function

The three necessary conditions (5), (6) and (7) characterize the pre sign-up optimal contract. For our functional specifications, we get $\mu = 1$ from (7). Using this in (6), we obtain

$$\dot{\lambda} = \frac{d\lambda}{dn} = -f\frac{-1}{1} - \lambda \cdot 0 - 1 \cdot f = 0$$

This implies that $\lambda(n) = \lambda$, a fixed constant. From the transversality condition $\lambda(n_H) = 0$, we get $\lambda = 0$. Reinserting $\lambda = 0$ in (5),

$$f \cdot Y_u = 0 \Rightarrow Y_u = 0$$

Evaluating Y_u:

$$Y_u = \frac{\partial}{\partial u}(v_1(n) + v_2 - c(u - n)) = 0$$
$$-c' = 0$$

This necessitates that $c(u-n)$ is constant. Further, since $c(0) = 0$, we get $c(u-n) = c(0)$ which yields $u(n) = n$. Moreover, since $\mu = 1$ this implies that the inequality (3) binds. Using $Y = v_1(n) + v_2 - 0$ and from the incentive compatibility constraint:

$$\int_{n_L}^{n_H} (v_1(n) + v_2) f(n) dn = \bar{Y}$$

we obtain $v_1(n) = 0$ and $v_2 = \bar{Y}$.

C Proof of Proposition 3

We note that the post sign-up participation constraint is slack and hence the corresponding multiplier $\mu = 0$. Using this in equation (6), we obtain:

$$\frac{d\lambda}{dn} = f(n)$$

which along with the transversality condition $\lambda(n_L) = 0$ yields:[9]

$$\lambda(n) = F(n)$$

where $F(n) = \int_{n_L}^{n} f(t)dt$. Using this value of the costate variable λ in equation (5) we obtain,

$$\frac{c'(u-n)}{c''(u-n)} = \frac{F(n)}{f(n)} \tag{18}$$

[9]This transversality condition is different from the pre-sign up case and features a vertical line boundary condition corresponding to the initial state $Y(n_L) \geq \bar{Y}$ (Chiang, 1992, chapter 3).

At the realization $n = n_L$, since $F(n_L) = 0$, it must be that $c'(u(n_L) - n_L) = 0 = c'(0)$, which immediately implies that $u(n_L) = n_L$. Additionally for any $n \neq n_L$, since the right hand side is positive, it implies that $c'(u(n) - n) > c'(0)$ which yields:

$$u(n) > n \quad \forall n \in (n_L, n_H] \text{ and } u(n_L) = n_L \tag{19}$$

Finally, noting that

$$\int_{\bar{Y}}^{Y} dY = \int_{n_L}^{n} \frac{dY}{dn} dn = \int_{n_L}^{n} \frac{\partial Y}{\partial n} dn$$

$$Y = \bar{Y} + \int_{n_L}^{n} c'(u(t) - t)dt \tag{20}$$

$$v_1(n) + v_2 = \bar{Y} + c(u(n) - n) + \int_{n_L}^{n} c'(u(t) - t)dt \tag{21}$$

In particular, equation (21) suggests that the advertiser pay the agent a fixed payment of $v_2 = \bar{Y}$ and a variable payment equaling $v_1(n) = c(u(n) - n) + \int_{n_L}^{n} c'(u(t) - t)dt$.

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